

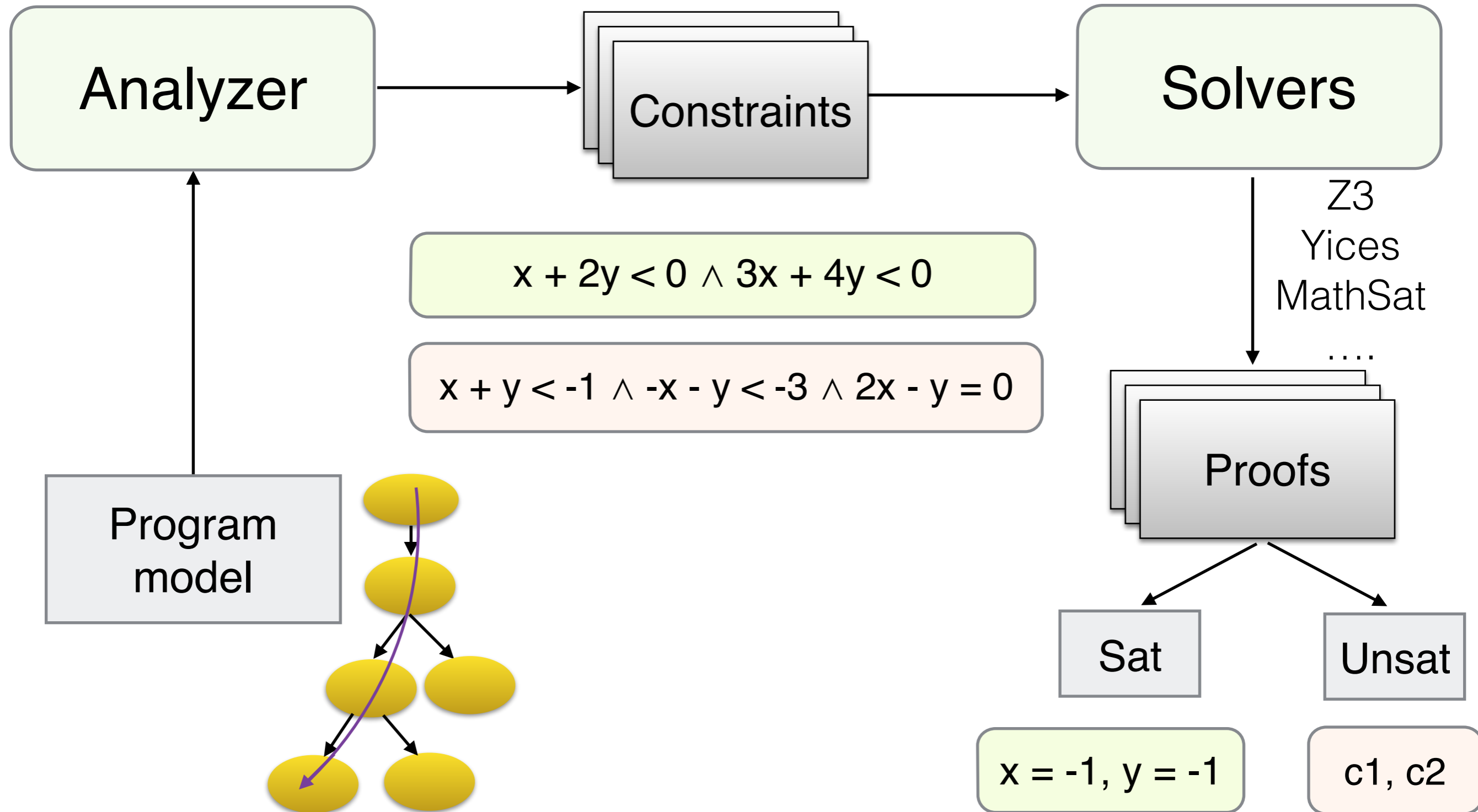
Reusing Constraint Proofs in Program Analysis

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Giovanni Denaro⁺, Mauro Pezzè^{*,+}

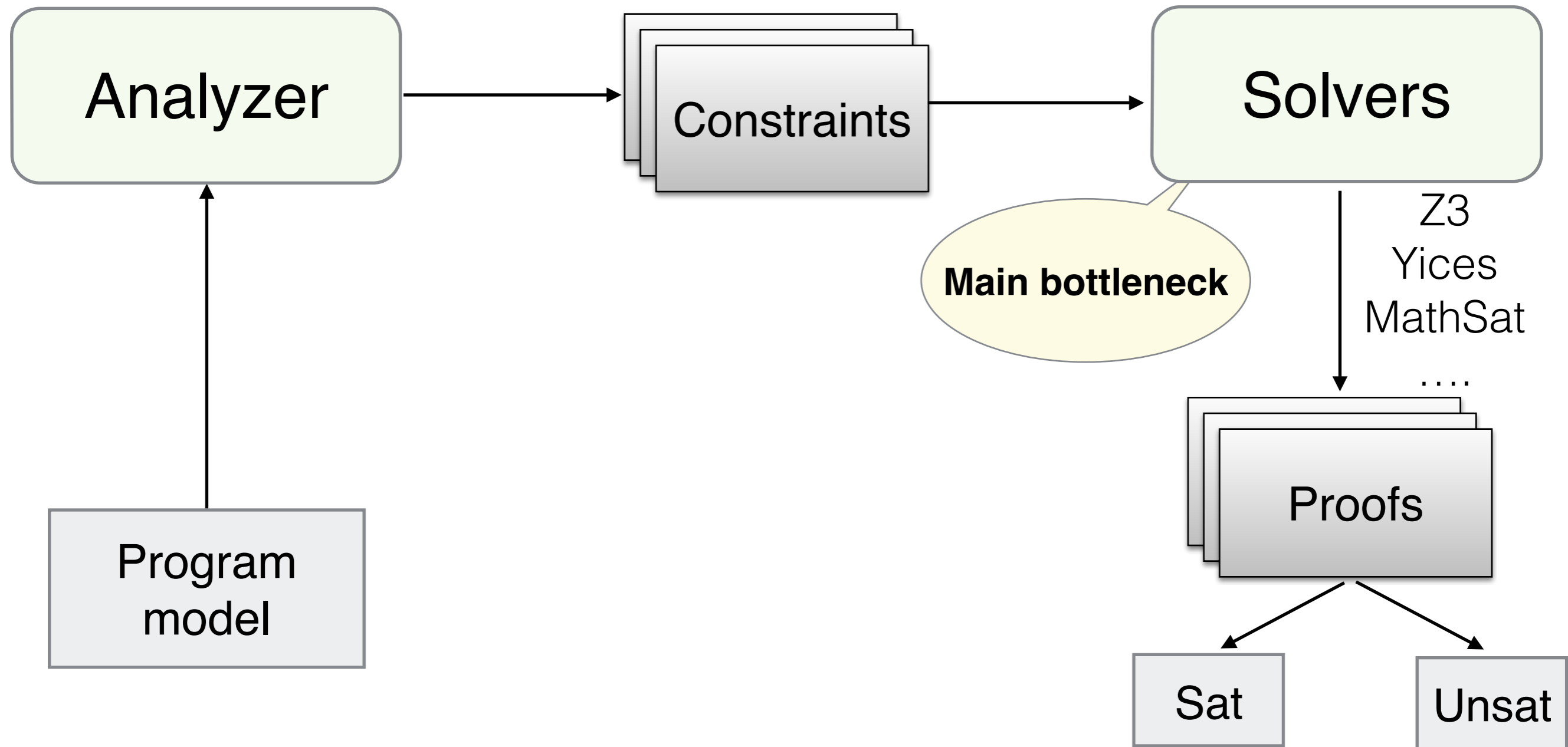
**Università della Svizzera italiana (USI),
Switzerland*

*+ University of Milano-Bicocca,
Italy*

Program Analysis



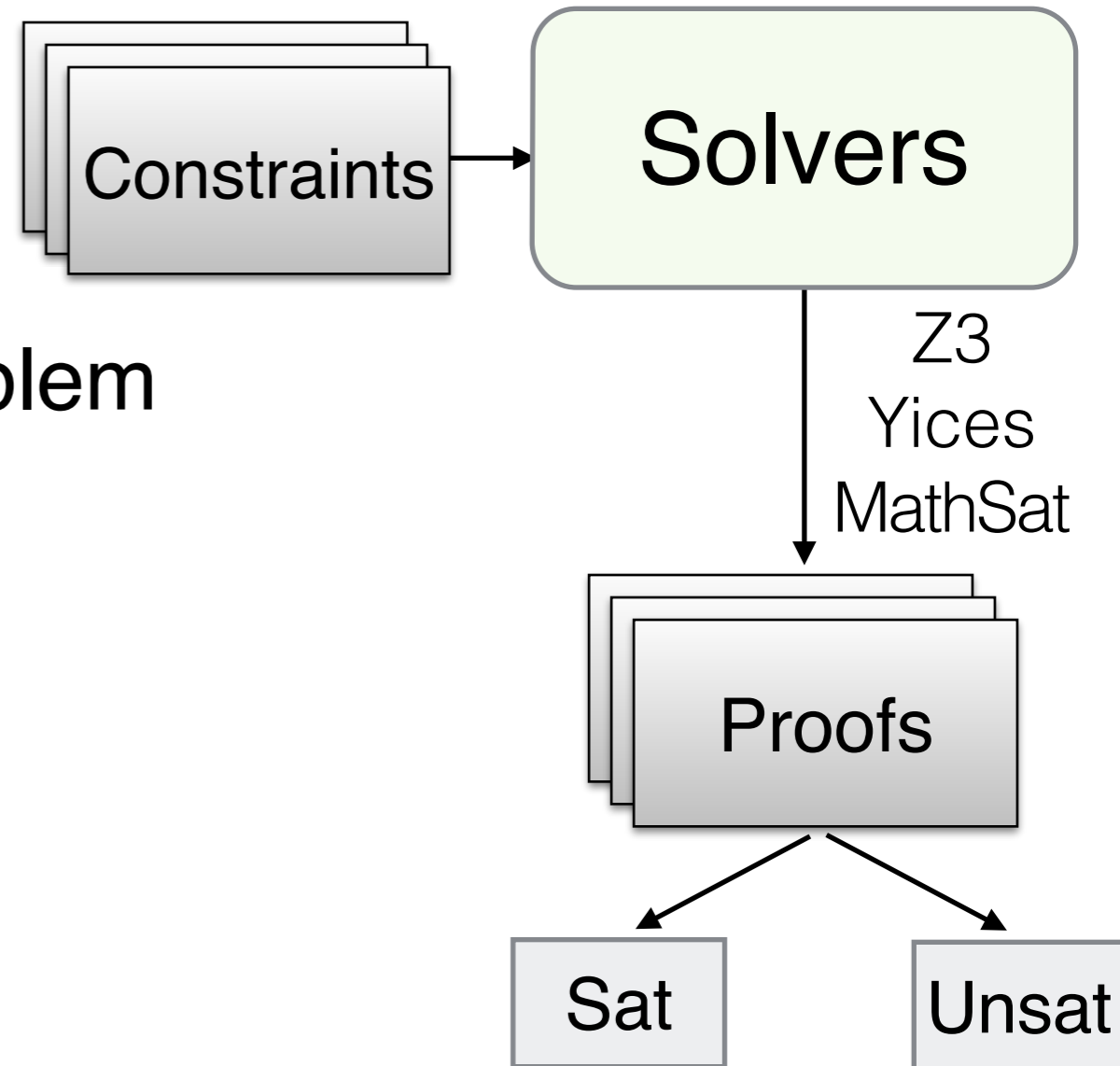
Main Bottleneck



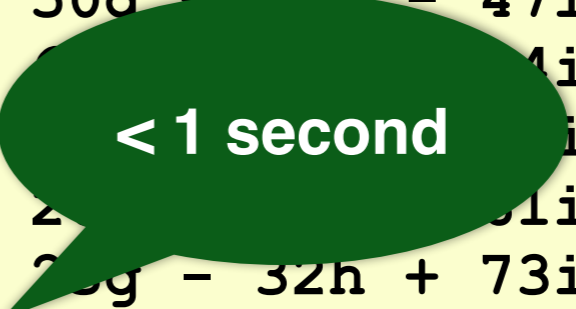
Solving time accounts for 92% of overall execution time on average. (KLEE. Cadar et al. osdi'08)


Main Bottleneck

- High complexity of the SMT problem
- A large set of big constraints
- Solving time hard to predict

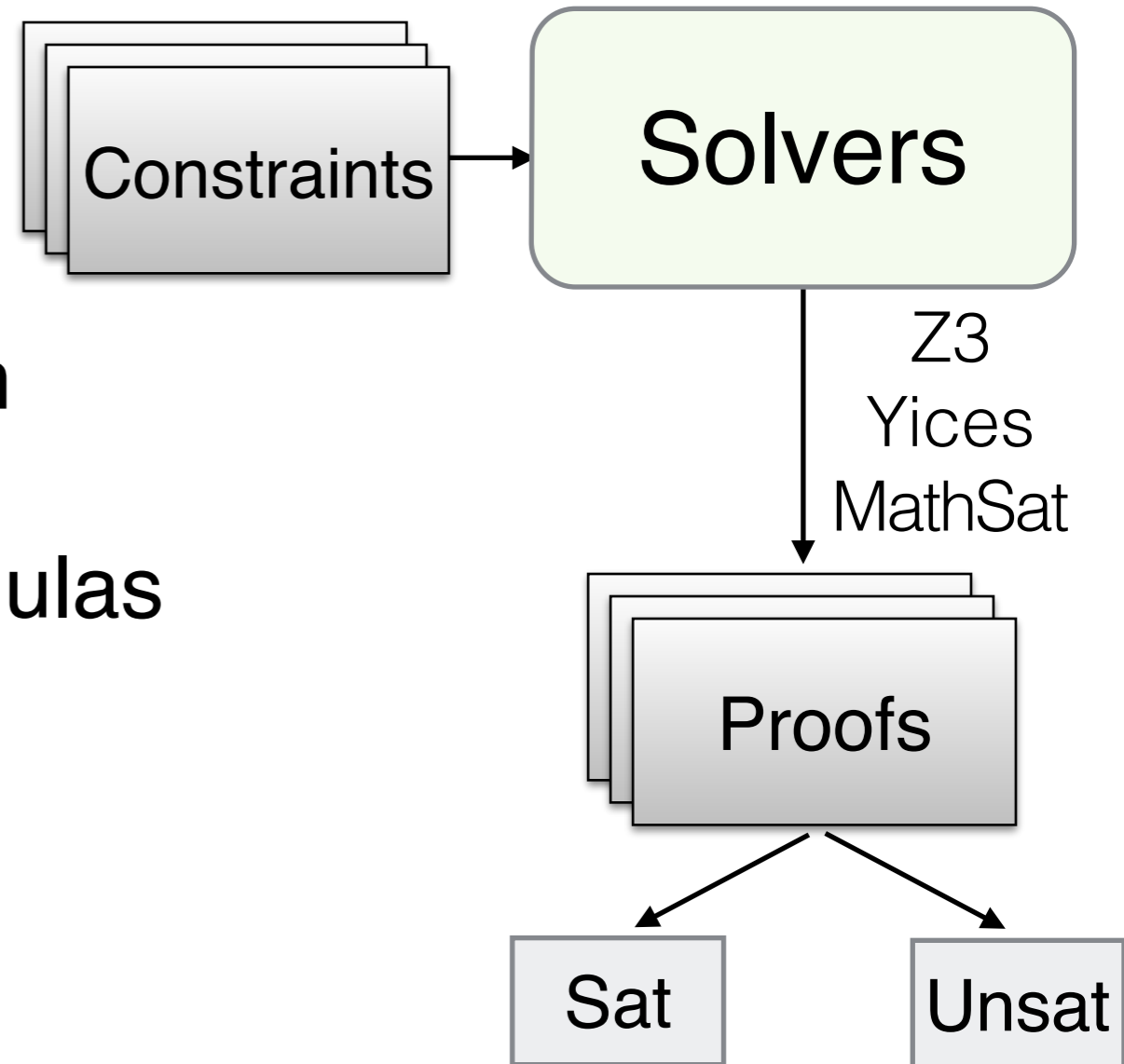


Solving time is hard to predict

$$\begin{array}{r} -2a + 85b - 90c - 44d + 39e + 96f - 76g - 88h - 72i - 79j \leq 66 \\ -100a - 19b + 60c - 96d - 42e - 30f + 82g + 75h + 73i - 41j \leq 97 \\ -56a + 96b - 15c - 45d - 33e - 42f + 50g + 9h - 47i - 92j \neq 64 \\ 41a + 79b + 9c - 96d - 35e + 24f - 8g + 6h - 4i - 58j \neq 41 \\ -67a - 65b - 46c - 49d + 71e + 100f - 2g + 5h - 3i + 64j \leq 48 \\ -80a + 59b + 95c - 4d + 32e + 39f + 2g + 1h - 5i + 35j \leq 32 \\ 68a + 70b + 66c - 43d + 32e - 69f + 2g - 32h + 73i - 28j \neq 12 \\ -45a + 51b - 88c - 46d - 27e + 9f + 34g + 57h + 14i - 1j \neq 60 \\ -52a - 46b + 55c - 74d - 21e - 52f - 55g + 41h - 96i + 61j \leq 9 \\ 53a + 68b + 3c + 15d + 50e - 38f + 25g - 82h - 96i + 11j \leq 9 \end{array}$$


$$\begin{array}{r} 54a + 90b - 32c + 45d - 73e + 77f - 98g + 54h - 45i - 67j \neq 4 \\ 52a + 22b + 71c + 40d + 21e - 75f - 75g + 13h + 33i - 18j \leq 12 \\ -17a - 100b + 56c - 94d + 79e + 19f + 39g - 53h - 78i + 98j \leq 2 \\ -38a + 72b - 86c - 8d + 54e - 68f + 44g + 5h + 34i + 72j \leq 81 \\ 66a - 73b + 86c - 44d - 66e + 22f + 9g + 1h - 91j \leq 37 \\ -51a - 64b - 19c + 80d - 74e + 37f - 8g + 6h - 30j \neq 44 \\ 71a - 44b + 3c - 4d + 14e - 18f + 15g + 5h - 60j \neq 91 \\ -89a + 4b - 73c + 5d + 39e + 4f + 8g - 2h - 16i + 95j \neq 37 \\ 13a + 56b + 87c - 39d - 60e - 36f + 35g + 74h - 3i + 5j \leq 70 \\ -37a + 51b - 30c + 24d + 34e + 63f + 84g - 34h + 91i + 39j \neq 66 \end{array}$$


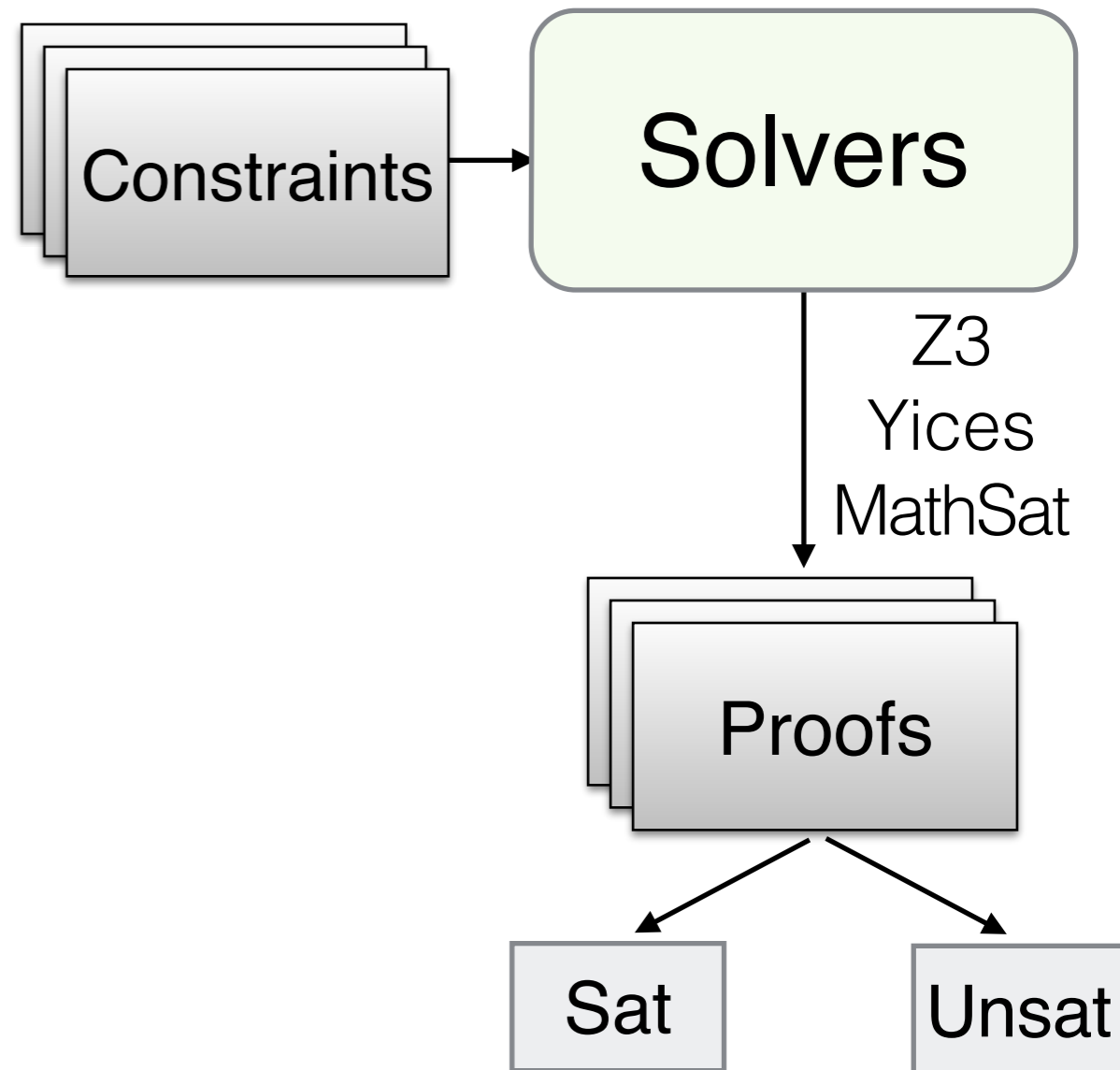
Main Bottleneck



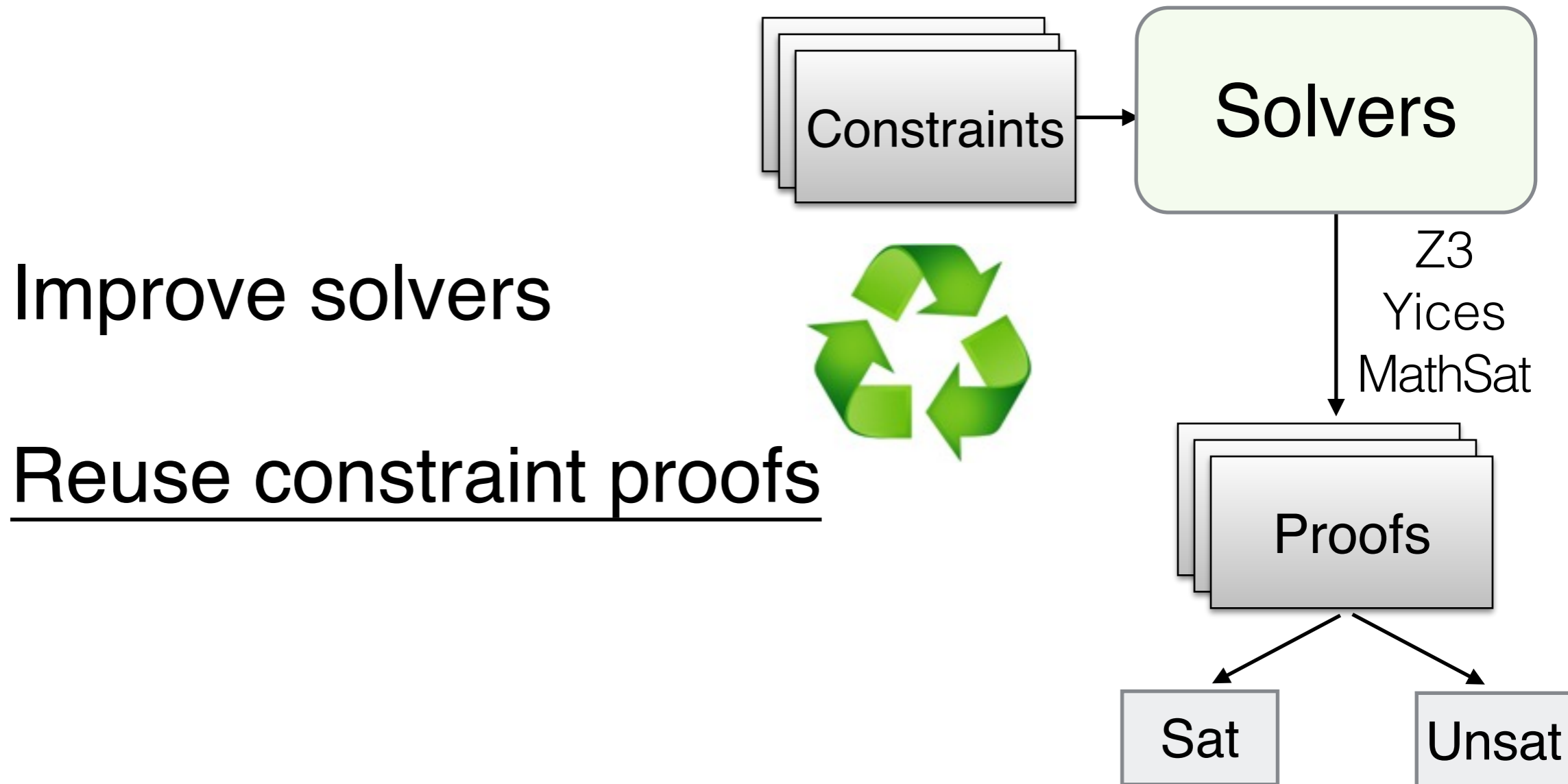
- High complexity of SMT problem
- A large set of big constraint formulas
- Solving time hard to predict

Overcome the Bottleneck

Improve solvers



Overcome the Bottleneck



Reuse Proofs

$$x + y < 0 \wedge a + 2b \neq 9 \wedge x - y \neq 2 \wedge a - b > 10$$

$$x + y \geq 0 \wedge x - y = 2 \wedge a + 2b \neq 9 \wedge a - b > 10$$

Reuse Proofs

$$x + y < 0 \wedge \mathbf{a + 2b \neq 9} \wedge x - y \neq 2 \wedge \mathbf{a - b > 10}$$

$$x + y \geq 0 \wedge x - y = 2 \wedge \mathbf{a + 2b \neq 9} \wedge \mathbf{a - b > 10}$$

Slicing

$$x + y < 0 \wedge x - y \neq 2$$

$$\mathbf{a + 2b \neq 9} \wedge \mathbf{a - b > 10}$$

$$x + y \geq 0 \wedge x - y = 2$$

$$\mathbf{a + 2b \neq 9} \wedge \mathbf{a - b > 10}$$

State of the Art

KLEE

(OSDI'08, Cadar et al.)

GREEN

(FSE'12, Visser et al.)

Slicing

Simplification

Variable
renaming

Improve the State of the Art



KLEE

(OSDI'08, Cadar et al.)

GREEN

(FSE'12, Visser et al.)

Recognize More Reusable Constraints

(1) Equivalence by reordering terms and clauses

(2) Stricter constraints by containment and implication

(1) Equivalence by reordering terms and clauses

C₁ $x + 2y + 1 < 0 \wedge$
 $3x + 4y - 1 < 0$

$2y + x + 1 < 0 \wedge$
 $4y + 3x - 1 < 0$

$4y + 3x - 1 < 0 \wedge$
 $2y + x + 1 < 0$

C₂ $4a + 3b - 1 < 0 \wedge$
 $2a + b + 1 < 0$

$4V_1 + 3V_2 - 1 < 0 \wedge$
 $2V_1 + V_2 + 1 < 0$

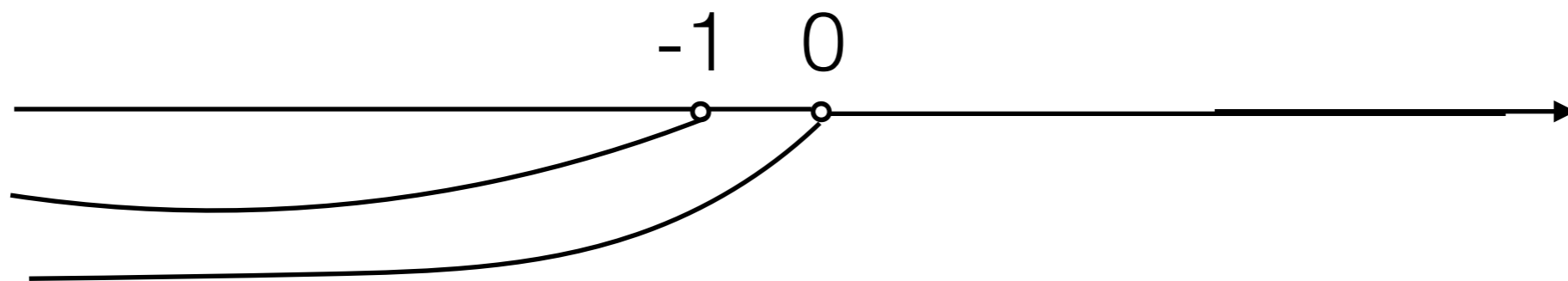
(2) Stricter constraints by containment and implication

C1:

$$X < -1$$

C2:

$$X < 0$$



Our Solution

(1) Equivalence by reordering terms and clauses

(2) Stricter constraints by containment and implication

(1) Equivalence by reordering terms and clauses

$$C_1 \equiv C_2 \text{ iff } C_1 \in \text{Permutation}(C_2)$$

Permutation-based Equivalence Problem = Graph Isomorphism Problem

Search for equivalent constraints?

Equivalent Constraints Search

via Canonical Form

$$C_1 \equiv C_2 \Leftrightarrow \text{canonical}(C_1) = \text{canonical}(C_2)$$

Equivalent Constraints Search

via Canonical Form

$$C_1 \equiv C_2 \Leftrightarrow \text{canonical}(C_1) = \text{canonical}(C_2)$$

C₁

$$x + 2y + 1 \leq 0 \wedge \\ 3x + 4y - 1 \leq 0$$

1	2	1	\leq
3	4	-1	\leq

C₂

$$4a + 3b - 1 \leq 0 \wedge \\ 2a + b + 1 \leq 0$$

4	3	-1	\leq
2	1	1	\leq

Canonical form

4	3	-1	\leq
2	1	1	\leq

The Canonicalization Algorithm

$$\begin{aligned} &2a + b \leq 0 \\ \wedge &a + 2b \leq 0 \\ \wedge &a \neq 0 \\ \wedge &a + 3b \leq 0 \\ \wedge &a - 1 \leq 0 \end{aligned}$$



2	1	0	\leq
1	2	0	\leq
1	0	0	\neq
1	3	0	\leq
1	0	-1	\leq

The Canonicalization Algorithm

sort rows by comparison and constant terms

2	1	0	\leq
1	2	0	\leq
1	0	0	\neq
1	3	0	\leq
1	0	-1	\leq

The Canonicalization Algorithm





sort rows by comparison and constant terms

2	1	0	\leq
1	2	0	\leq
1	0	0	\neq
1	3	0	\leq
1	0	-1	\leq

The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

				
2	1	0	\neq	
1	2	0	\neq	
1	3	0	\neq	
1	0	<u>-1</u>	\neq	
1	0	<u>0</u>	\neq	

0

initial

0

1-D locked






0

2-D locked

The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

				
1	3	<u>0</u>	≠	
2	1	0	≠	
1	2	0	≠	
1	0	<u>-1</u>	≠	
1	0	<u>0</u>	≠	

0

initial

0

1-D locked

0








2-D locked

The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

sort 1-D-locked rows and columns lexicographically

				
<u>3</u>	<u>1</u>	<u>0</u>	\leq	
1	2	0	\leq	
2	1	0	\leq	
<u>0</u>	<u>1</u>	<u>-1</u>	\leq	
<u>0</u>	<u>1</u>	<u>0</u>	\neq	

0

initial

0

1-D locked

0










2-D locked

The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

sort 1-D-locked rows and columns lexicographically

				
<u>3</u>	<u>1</u>	<u>0</u>	<u>≠</u>	
<u>2</u>	<u>1</u>	<u>0</u>	<u>≠</u>	
<u>1</u>	<u>2</u>	<u>0</u>	<u>≠</u>	
<u>0</u>	<u>1</u>	<u>-1</u>	<u>≠</u>	
<u>0</u>	<u>1</u>	<u>0</u>	<u>≠</u>	

0

initial

0

1-D locked

0

2-D locked





The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

sort 1-D-locked rows and columns lexicographically

sort the remaining rows and columns by brute-force

						
4	4	4	4	0	∞	
3	0	1	0	0	∞	
3	1	0	0	0	∞	
3	0	0	1	0	∞	

0

initial

0

1-D locked

0

2-D locked











The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

sort 1-D-locked rows and columns lexicographically

sort the remaining rows and columns by brute-force

						
<u>4</u>	<u>4</u>	<u>4</u>	<u>4</u>	<u>0</u>	<u>∞</u>	
<u>3</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>0</u>	<u>∞</u>	
<u>3</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>0</u>	<u>∞</u>	
<u>3</u>	<u>0</u>	<u>0</u>	<u>1</u>	<u>0</u>	<u>∞</u>	

0

initial

0

1-D locked

0

2-D locked

The Canonicalization Algorithm

sort rows by comparison and constant terms

sort rows and columns by biggest values

sort 1-D-locked rows and columns lexicographically

sort the remaining rows and columns by brute-force

Polynomial

93% of constraints converge up to the polynomial steps.

Exponential

(2) Stricter constraints by containment and implication

What is a stricter constraint?

Search for stricter constraints?

Stricter Constraints

C_1

$$3X < 0 \wedge \\ X + Y < 10$$

Sat

$$3X < 0 \wedge \\ X + Y < 10 \wedge \\ 2X - Y = 0$$

$$3X < \underline{-1} \wedge \\ X + Y < 10$$

Stricter Constraints

C₁

$$3X < 0 \wedge \\ X + Y < 10$$

Sat

$$3X < 0 \wedge \\ X + Y < 10 \wedge \\ 2X - Y = 0$$

$$3X < \underline{-1} \wedge \\ X + Y < 10$$

C₂

$$X + Y < -1 \wedge \\ -X - Y < -3 \wedge \\ 2X - Y = 0$$

UnSat

$$X + Y < -1 \wedge \\ -X - Y < -3$$

$$X + Y < \underline{0} \wedge \\ -X - Y < -3$$

Stricter Constraints Search

Clause-to-constraint index

Stricter Constraints Search

Clause-to-constraint index

C_0

Cache

C_1

C_2

C_3

$$3X < 0 \wedge X + Y < 10$$

$$3X < -1 \wedge X + Y < 10 \text{ (sat)}$$

$$3X < -1 \wedge X - 2Y < 0 \text{ (sat)}$$

$$-2X < -1 \wedge X + Y < 10 \text{ (sat)}$$

$$3X \longrightarrow \{C_1, C_2\}$$

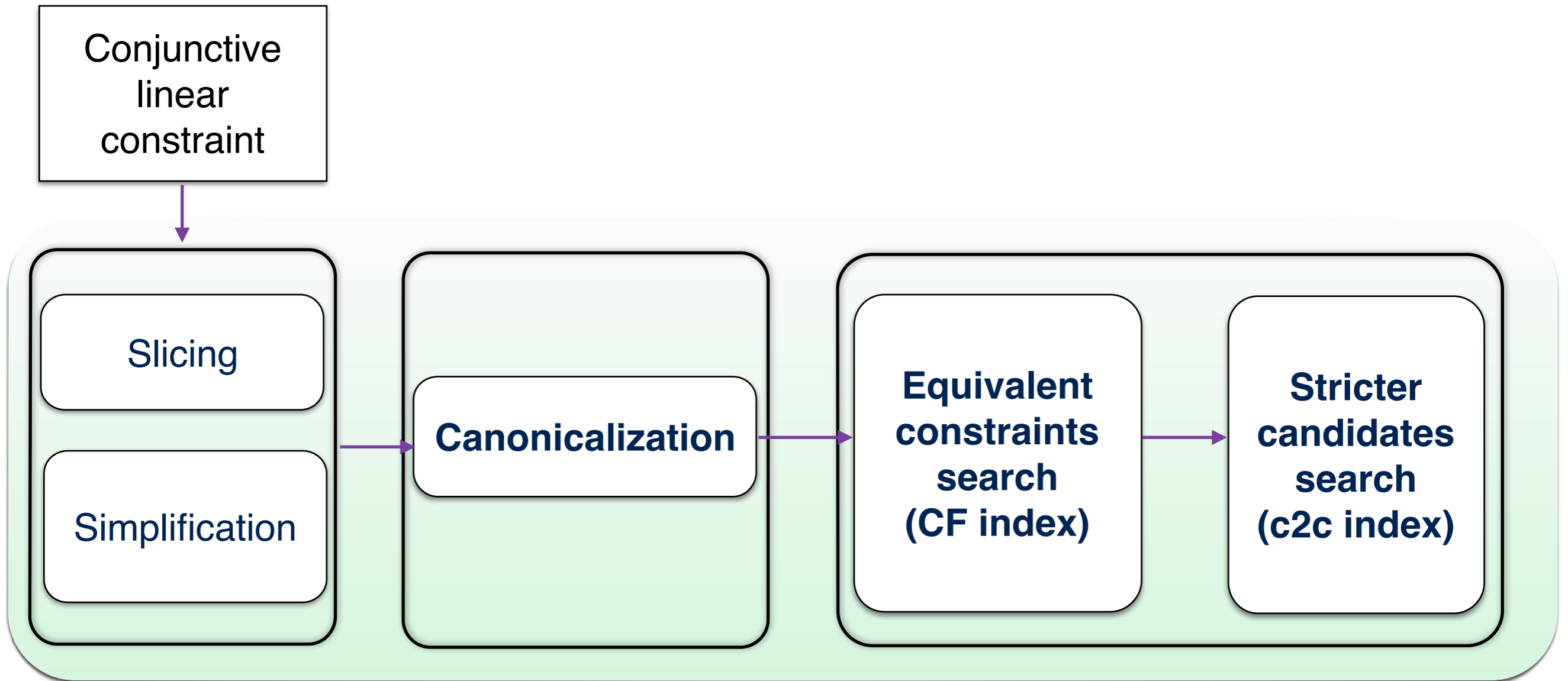
$$X + Y \longrightarrow \{C_1, C_3\}$$

intersection

$$\{C_1, C_2\} \cap \{C_1, C_3\} = \{C_1\}$$

The Recal Framework

The Recal Framework



Evaluation

Effectiveness: Can Recal **effectively** identify reusable constraints?

Efficiency: Is Recal more **efficient** than SMT solvers?

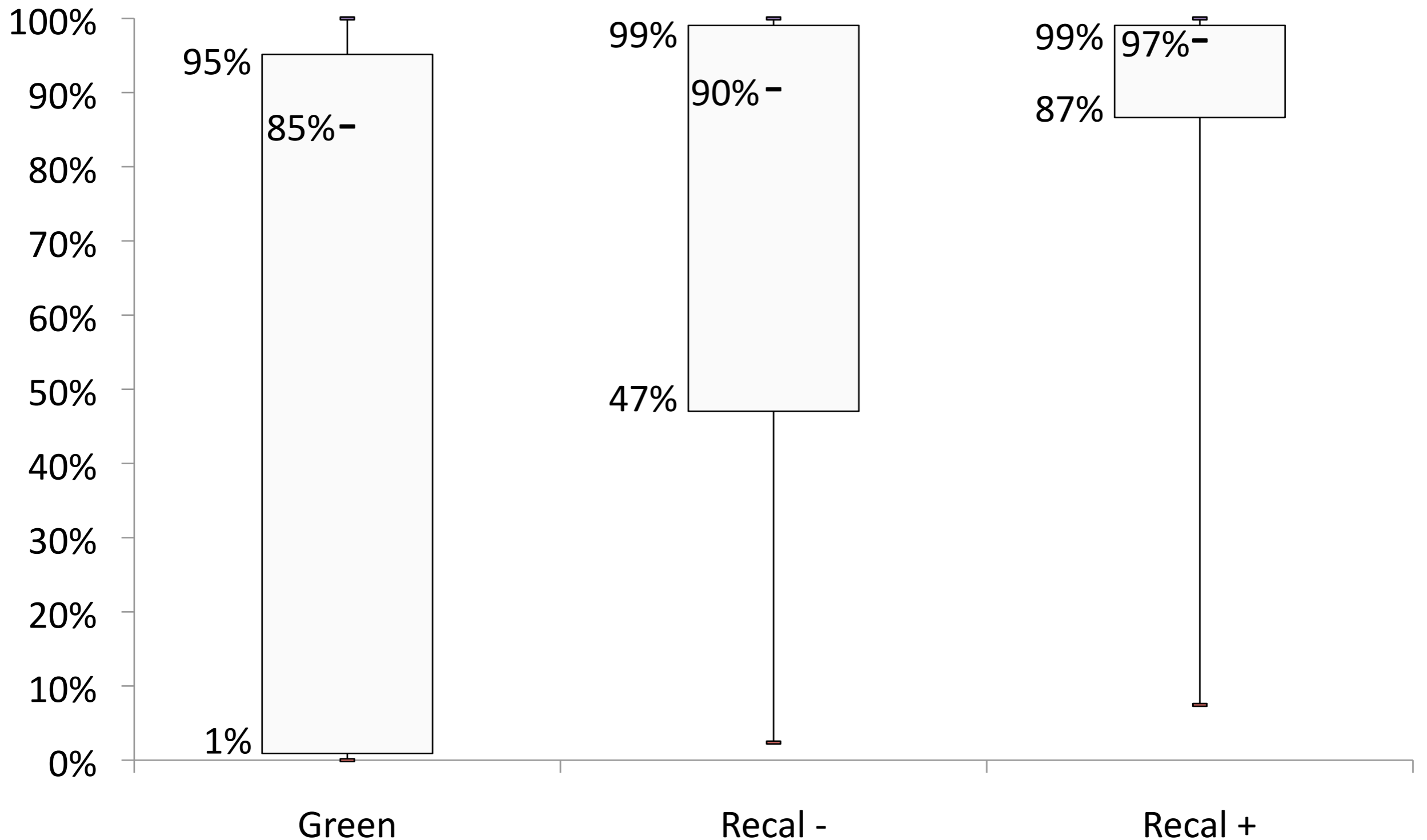
A large set of real-world constraints

Program	LOC	Language	#Constraints
old-tax	78	Java	27
new-tax	78	Java	35
afs	75	Java	48
dijkstra	142	Java	85
doubly-linked-list	806	Java	114
swapwords	30	Java	173
kbfiltr	599	C	188
wbs	297	Java	191
ball	15	Java	202
reverseword	32	Java	303
block	79	Java	336
division	87	Java	1,257
multiplication	50	Java	1,263
collision	21	Java	1,741
avl	519	Java	2,985
tcas	200	Java	9,780
treemap	806	Java	13,556
cdaudio	2171	C	55,329
floppy	1137	C	100,006
grep	10068	C	100,126
diskperf	1104	C	103,505

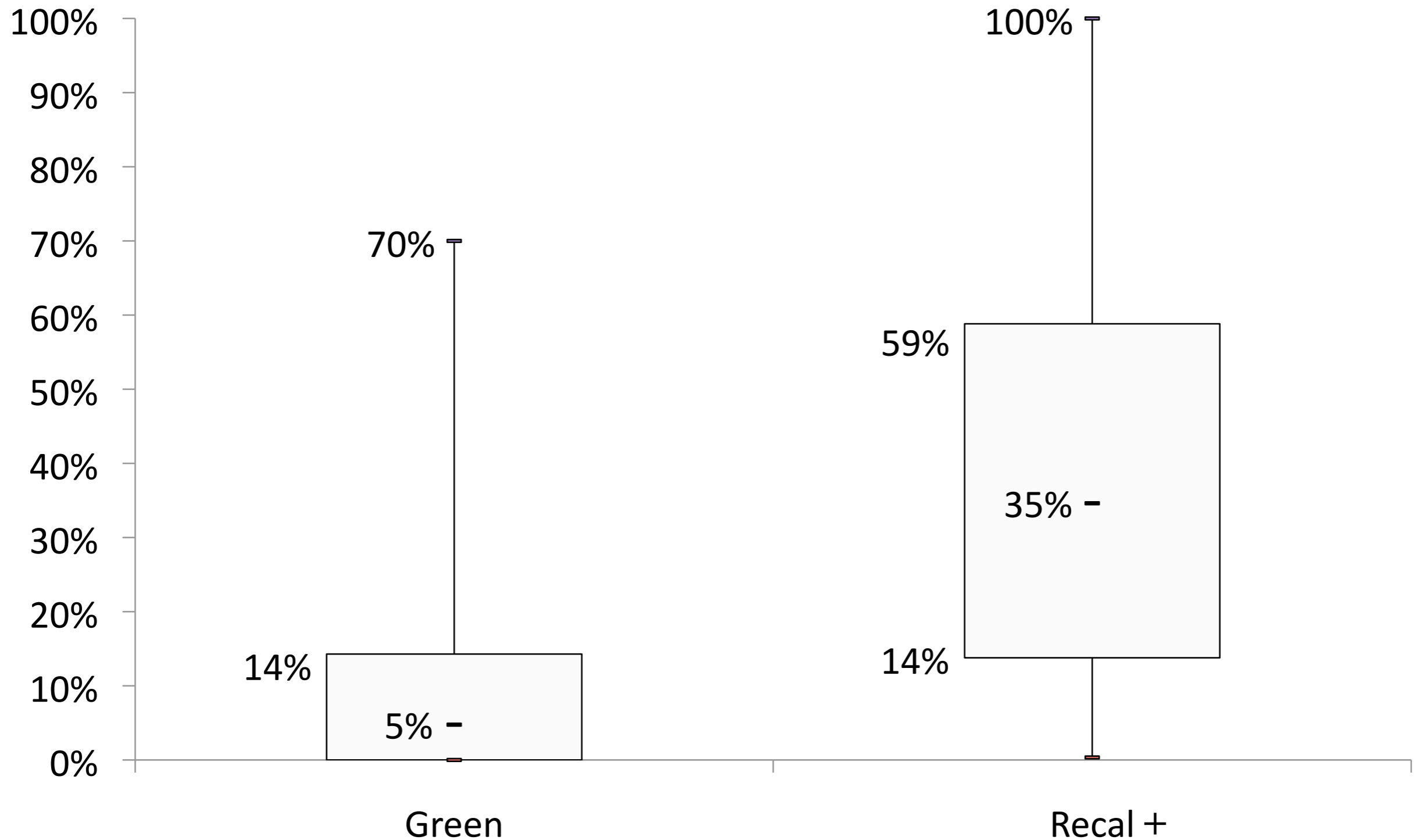
JBSE [Braione, et al., FSE'13]
CREST [Burnim, et al., EECS'08]

Constraints
391,250

Intra-program Reuse Rates



Inter-program Reuse Rates



High Reuse Rates

Program	LOC	Language	#Constraints
old-tax	78	Java	27
new-tax	78	Java	35
afs	75	Java	48
dijkstra	142	Java	85
doubly-linked-list	806	Java	114
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kbfs			88
wh			91
ba			92
rev			93
bl			96
div			97
multiplication	50	Java	1,263
collision	21	Java	1,741
avl	519	Java	2,985
tcas	200	Java	9,780
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Formulas: 391,250

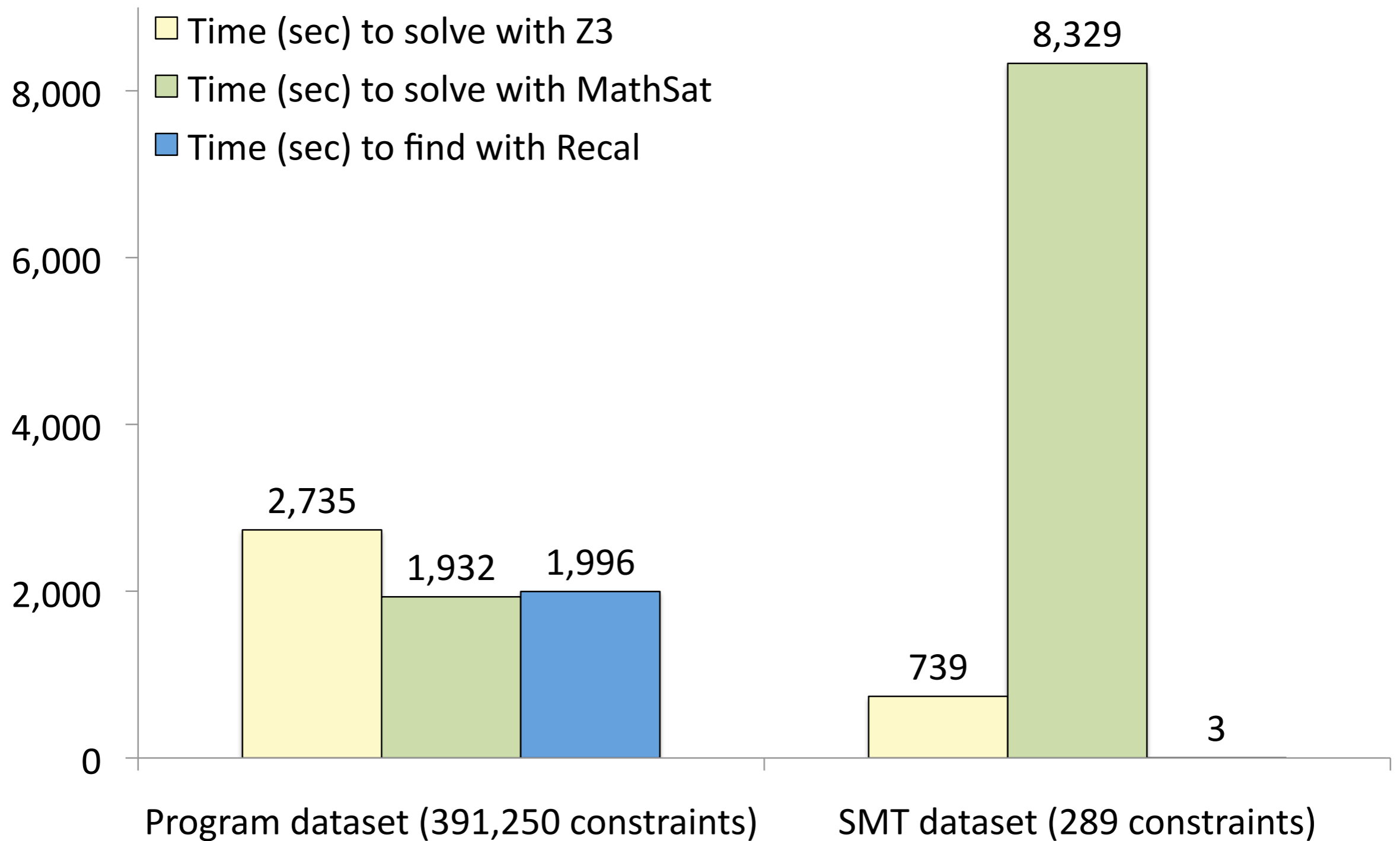
Queries to Solver: ~1,010

Evaluation

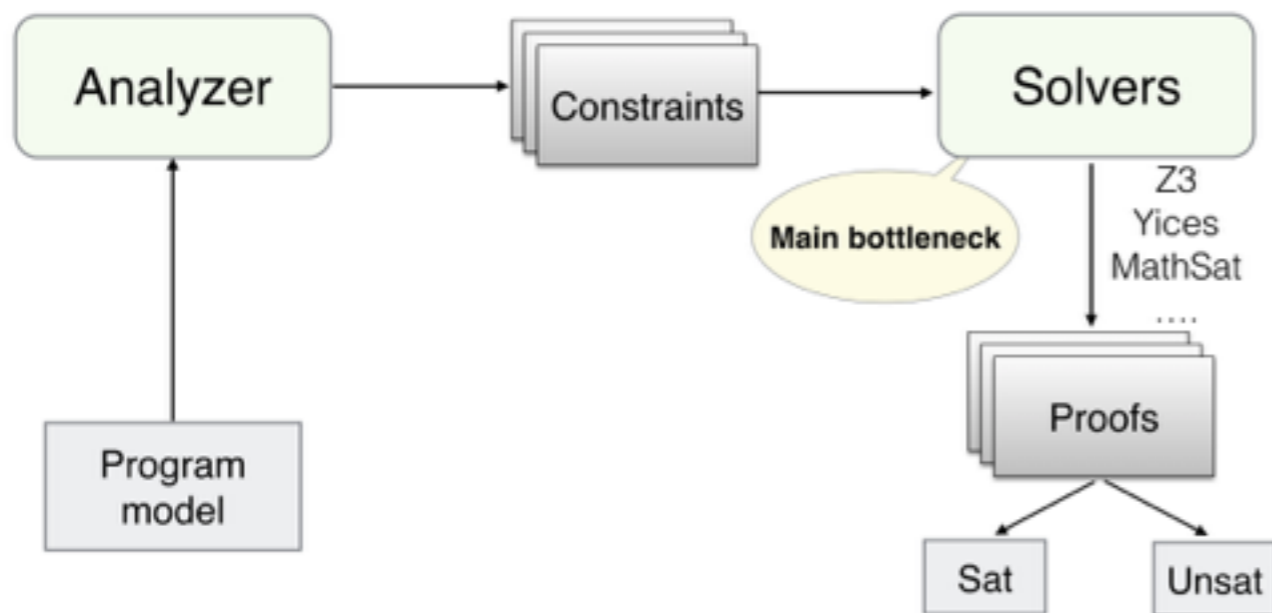
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Efficiency: Is Recal more **efficient** than SMT solvers?

Searching vs. Solving



Main Bottleneck



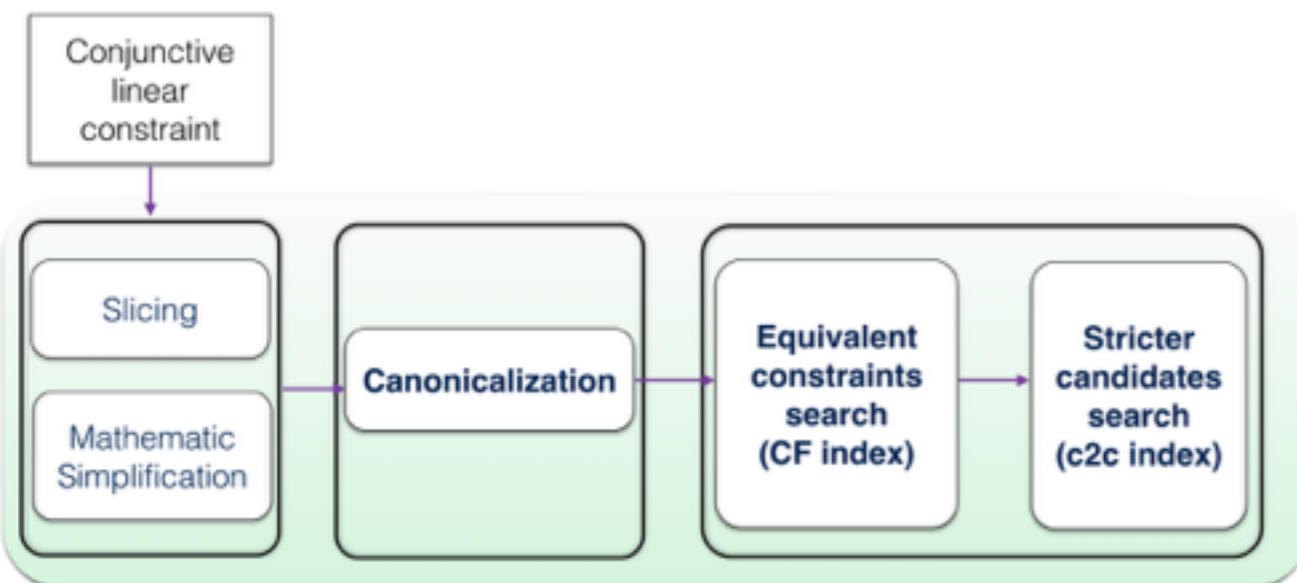
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Searching vs. Solving

